# The twist-2 Compton operator and its hidden Wandzura-Wilczek and Callan-Gross structures

Bodo Geyer
*Institute of Theoretical Physics, Leipzig University, Augustusplatz 10, D-04109 Leipzig, Germany*

Dieter Robaschik
*BTU Cottbus, Fakultät 1, Postfach 101344, D-03013 Cottbus, Germany*
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Abstract: Power corrections for virtual Compton scattering at leading twist are determined at operator level. From the complete off-cone representation of the twist-2 Compton operator integral representations for the trace, antisymmetric and symmetric part of that operator are derived. The operator valued invariant functions are written in terms of iterated operators and may lead to interrelations. For matrix elements they go over into relations for generalized parton distributions. – Reducing to the $s$-channel relevant part one gets operator pre-forms of the Wandzura-Wilczek and the (target mass corrected) Callan-Gross relations whose structure is exactly the same as known from the case of deep inelastic scattering; taking non-forward matrix elements one reproduces earlier results [1] for the absorptive part of the virtual Compton amplitude. – All these relations, obtained without any approximation or using equations of motion, are determined solely by the twist-2 structure of the underlying operator and, therefore, are purely of geometric origin.

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## I. INTRODUCTION

Virtual Compton scattering is an excellent example for sharpening our understanding of hadron physics both theoretically and experimentally. Already in the early days of QCD, studying deep inelastic scattering (DIS), many remarkable results were obtained either using simple, approximate models, also phenomenological ones, or applying general quantum field theoretical methods. Among these results are the well-known Callan-Gross (CG) [2] and the Wandzura-Wilczek (WW) relations [3] for the structure functions in the unpolarized and polarized DIS, respectively, and the enlightenment of the influence of target masses [4, 5, 6]. The extension of these results to deeply virtual Compton scattering (DVCS), requiring the consideration of non-forward matrix elements of the Compton operator, appeared to be more difficult – not only because of the harder computations but also for conceptual reasons. The latter are connected with the necessary use of generalized (multi-parameter) distribution amplitudes – usually called generalized parton distributions (GPD) – and with their twist decomposition.

As is well known, the twist decomposition of non-scalar non-local QCD operators, especially off the light-cone, is highly nontrivial. Moreover, it suffers from the fact that the notion of twist is not unique: twist is defined either group theoretically for the non-local operators and denoted as geometric twist [7] by extending the original definition for local operators [8] or, differently, it is phenomenological determined for the individual kinematic invariants of the complete scattering process [9] counting powers of $1/Q^2$. Both definitions coincide only at leading order. In the past, investigating the group theoretic approach many of its general aspects have been discovered and generically applied to the relevant light-cone dominated hard scattering processes. Within that frame the various higher twist GPDs are related to matrix elements of non-local operators of higher (geometric) twist.

Recently, studying the target mass corrections of the virtual non-forward Compton scattering at leading twist a generalization of the WW- and the (target mass corrected) CG-relations has been found which have exactly the same formal structure as in the forward case [1]. This result has been obtained using only the complete geometric twist-2 structure off-cone, i.e., without applying the equations of motion. Analogous observations have been made earlier when – extending the consideration of deep inelastic scattering at leading twist [10] – the parton distributions at higher twist [11], deep inelastic scattering with an additional meson in the final state [12], diffractive scattering [13] as well as the behaviour of vector meson distribution amplitudes [14] had been considered. These rather interesting results led to the conjecture [1] that it should be the outcome of the very structure of the twist-2 Compton operator itself. If so, the observed relations should follow from matrix elements of appropriate operator relations and, moreover,

would hold also for other light-cone dominated processes, e.g., with other particles in the final state which, in fact, is of phenomenological importance.

The aim of the present paper is to extract the corresponding operator relations. Thereby, the above mentioned processes are to be distinguished insofar that in the case of $s$-channel processes, like DIS or DVCS, besides the investigation of the entire operator expression – whose matrix elements correspond to the scattering amplitudes – the absorptive part can be discussed separately. Of course, also the amplitudes of light-cone dominated $t$-channel processes, like electro-production of hadrons and hadron wave functions, are determined by matrix elements of the entire operator expressions. However, in that case a separation of the absorptive part at operator level remains an open problem, at least presently. In fact, we first consider the structure of the entire (twist-2) Compton operator and its power behavior. Thereafter, we concentrate on that expression which is relevant for the $s$-channel absorptive part and we derive its general operator relations.

In the next Section, for the sake of notation, we remind the generic phenomenological situations where the Compton operator appears and introduce the appropriate approximations as well as point to its geometric twist decomposition off-cone. After that, in three separate Sections we consider the trace part $\widehat{T}^{\rm tw2}_{\rm trace}(q)$, the antisymmetric part $\widehat{T}^{\rm tw2}_{[\mu\nu]}(q)$ and the symmetric part $\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q)$ of the Compton operator, respectively. By studying the trace part we already introduce the general procedure of the investigation. The explicit expressions of the corresponding Fourier transforms are taking over from our previous work [1]. Then, adapting the construction of corresponding sets of generalized distribution amplitudes $\Phi^{(i)}_a$ of Ref. [1], we introduce appropriate sets of iteratively defined operators $\mathcal{O}^{(i)}_\alpha$ of order $i$ which allow to obtain a representation of the various parts of the Compton operator in terms of independent kinematic invariants. At this stage also some observations concerning relations for the entire Compton operator are made. Then, we consider the operator version of the absorptive parts, denoted by ${\rm Im}\,\widehat{T}^{\rm tw2}_{\rm trace}(q)$, ${\rm Im}\,\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q)$ and ${\rm Im}\,\widehat{T}^{\rm tw2}_{[\mu\nu]}(q)$, respectively. We introduce an analogue of the Nachtmann variable together with a scaling variable which allows to rewrite these expressions as integrals over appropriate operator valued structure functions. The latter are completely analogous to the well-known structure functions occurring in DIS and their generalization to the non-forward case. In the final, concluding Section we summarize, point to open questions and comment on related work.

## II. BRIEF REVIEW OF PREREQUISITES AND RELATION TO PREVIOUS WORK

First, let us present two different generic processes (at leading twist-2) where the Compton operator appears and remember its amplitudes and its kinematics (spin dependence suppressed):
For the $s$-channel we take virtual Compton scattering, $\gamma^*(q_1) + h(P_1) \to \gamma^*(q_2) + h(P_2)$, whose amplitude reads

$$ {}^{s}T^{\rm tw2}_{\mu\nu}(q;P_i) = \langle P_2| \Big[\int {\rm d}^4x\; {\rm e}^{{\rm i}qx}\; \widehat{T}^{\rm tw2}_{\mu\nu}(x)\Big] |P_1\rangle\,, \tag{1} $$

where $P_1$ and $P_2$ ($q_1$ and $q_2$) are the momenta of the incoming and outgoing hadrons (virtual photons), respectively, $q = (q_2 + q_1)/2$, $p_+ = P_1 + P_2$ and $p_- = P_2 - P_1 = q_1 - q_2$ denotes the momentum transfer. By crossing, in the $t$-channel, we get electro production of hadron pairs, $\gamma^*(q_1) + \gamma^*(q_2) \to h(P_1) + h(P_2)$, whose amplitude reads

$$ {}^{t}T^{\rm tw2}_{\mu\nu}(q;P_i) = \langle\, 0\,| \Big[\int {\rm d}^4x\; {\rm e}^{{\rm i}qx}\; \widehat{T}^{\rm tw2}_{\mu\nu}(x)\Big] |P_1,P_2\rangle\,, \tag{2} $$

where $q_1$ and $q_2$ are the momenta of the incoming virtual photons and $P_1$ and $P_2$ are the momenta of the outgoing hadrons; again $p_\pm = P_2 \pm P_1$ but $q = (q_1 - q_2)/2$.

Both processes are considered in the generalized Bjorken region,

$$ qp_+ \longrightarrow \infty, \qquad Q^2 = -\,q^2 \longrightarrow \infty, \tag{3} $$

$$ \text{with}\qquad x_{\rm Bj} = Q^2/qp_+ \quad\text{and}\quad \eta = qp_-/qp_+ \qquad \text{fix}. \tag{4} $$

Thereby, the Compton operator in $x$-space is defined by

$$ \widehat{T}_{\mu\nu}(x) \;\equiv\; {\rm i}\,R\,T\,[\,J_\mu\,(x/2)\;J_\nu\,(-x/2\,)\,\mathcal{S}\,]\,, \tag{5} $$

where $J_\mu$ is the electromagnetic current, $R$ denotes the renormalization procedure and $\mathcal{S}$ is the (renormalized) $S-$matrix. In the generalized Bjorken region, the physical processes are dominated by the singularities of the Compton operator on the light-cone and, therefore, the operator product expansion can be applied. A general study of it, using the non-local light-cone expansion [15, 16], has been given, e.g., in Refs. [17, 18], cf. also Refs. [19, 20]. In leading order the Compton operator simply reads

$$ \widehat{T}_{\mu\nu}(x) \;\approx\; \frac{1}{2\,\pi^2\,(x^2-{\rm i}\epsilon)^2}\,\Big(S_{\mu\nu|}{}^{\alpha\beta}\,x_\alpha\,\mathcal{O}_\beta\,(\kappa x,-\kappa x) +\,\epsilon_{\mu\nu}{}^{\alpha\beta}x_\alpha\,\mathcal{O}_{5\,\beta}\,(\kappa x,-\kappa x)\Big)\,, \tag{6} $$

where the tensor $S_{\mu\nu|\alpha\beta} \equiv S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$ is symmetric in $\mu\nu$ and $\alpha\beta$ and $\kappa = 1/2$. (In order to omit the subtleties of operator mixing, the non-singlet case will be taken also suppressing the flavor structure.)

The hermitean, chiral-even vector and axial vector operators, $\mathcal{O}_\alpha(\kappa_1 x, \kappa_2 x)$ and $\mathcal{O}_{5\,\alpha}(\kappa_1 x, \kappa_2 x)$, respectively, are renormalized on the light-cone, $\tilde{x}^2 = 0$, first and then, in order to allow for an appropriate target mass expansion, extended off-cone by replacing $\tilde{x} \to x$. They are given by

$$\mathcal{O}_\alpha(\kappa_1 x, \kappa_2 x) \;=\; \mathrm{i}\big(O_\alpha(\kappa_1 x, \kappa_2 x) - O_\alpha(\kappa_2 x, \kappa_1 x)\big), \tag{7}$$
$$\mathcal{O}_{5\,\alpha}(\kappa_1 x, \kappa_2 x) \;=\; O_{5\,\alpha}(\kappa_1 x, \kappa_2 x) + O_{5\,\alpha}(\kappa_2 x, \kappa_1 x), \tag{8}$$

with

$$O_\alpha(\kappa_1 x, \kappa_2 x) \;:=\; RT\big[ :\bar{\psi}\,(\kappa_1\tilde{x})\,\gamma_\alpha\, U(\kappa_1\tilde{x}, \kappa_2\tilde{x})\;\psi\,(\kappa_2\tilde{x}) : \mathcal{S}\big]\big|_{\tilde{x}\to x}, \tag{9}$$
$$O_{5\,\alpha}(\kappa_1 x, \kappa_2 x) \;:=\; RT\big[ :\bar{\psi}\,(\kappa_1\tilde{x})\,\gamma_5\gamma_\alpha\, U(\kappa_1\tilde{x}, \kappa_2\tilde{x})\;\psi\,(\kappa_2\tilde{x}) : \mathcal{S}\big]\big|_{\tilde{x}\to x}. \tag{10}$$

The time-ordered phase factors, will be omitted in the following, i.e., the Schwinger-Fock gauge, $x^\mu A_\mu(x) = 0$, will be assumed. These definitions are chosen such that the generalized parton distributions (GPD) which follow by taking matrix elements would be real (analytic) functions.

Now, in the $s$-channel the absorptive part of the amplitude, $\mathrm{Im}\ {}^{s}T^{\mathrm{tw2}}_{\mu\nu}(q; P_i)$, may be considered which, after Fourier transformation of expression (6), results from the imaginary part of the quark propagator. This observation survives at the operator level. Therefore, we are able to construct an operator, denoted by $\mathrm{Im}\,\widehat{T}^{\mathrm{tw2}}_{\mu\nu}(q)$, whose matrix elements are the absorptive part of the amplitudes in the $s$-channel,

$$\langle P_2|\mathrm{Im}\ \widehat{T}^{\mathrm{tw2}}_{\mu\nu}(q)|P_1\rangle = \mathrm{Im}\ {}^{s}T^{\mathrm{tw2}}_{\mu\nu}(q; P_i)\,. \tag{11}$$

In fact, for that part of the amplitude of the virtual Compton scattering the WW- and (mass corrected) CG-relations were derived in Ref. [1]. They will be formulated here as operator relations. However, for $t$-channel processes the complete amplitude has to be considered. Despite of this, also in these cases, especially for meson distribution amplitudes as well as electro production of meson pairs, WW-like relations have been derived, either using equations of motion [21, 22, 23, 24] or using the notion of geometric twist alone [14].

Obviously, the operators (7) and (8) do not have definite twist – neither on-cone nor off-cone. The bi-local off-cone (axial) vector operators $O_{(5)\alpha}(\kappa_1 x, \kappa_2 x)$ contain contributions of any geometric twist $\tau = 2, 3, \ldots\infty$. The corresponding operators of definite twist recently have been determined in $x$–space [25]; by restriction on the light-cone one gets operators of geometric twist $\tau = 2, 3, 4$ which are known already from Ref. [7]. The general procedure which allowed to generalize the notion of geometric twist for the local operators [8] to the case of non-local ones [7], cf. also [17], consisted in expanding the non-local operators into a series of local ones, then decomposing these local operators with respect to irreducible representations of the Lorentz group and ordering them into towers of related operators of equal twist and, finally, summing up these towers into separate non-local operators again.

In the rest of the paper we restrict ourselves to the off-cone Compton operator of geometric twist 2,

$$\widehat{T}^{\mathrm{tw2}}_{\mu\nu}(x) = \frac{1}{2\pi^2(x^2 - i\epsilon)^2}\Big( S_{\mu\nu}{}^{\alpha\beta} x_\alpha \mathcal{O}^{\mathrm{tw2}}_{\beta}\,(\kappa x, -\kappa x) + \epsilon_{\mu\nu}{}^{\alpha\beta} x_\alpha \mathcal{O}^{\mathrm{tw2}}_{5\beta}\,(\kappa x, -\kappa x)\Big), \tag{12}$$

with [17]

$$O^{\mathrm{tw2}}_{(5)\alpha}\,(\kappa x, -\kappa x) = \sqrt{\pi}\,\partial_\alpha \int_0^1 d\tau \int \mathbf{d}^4 u\, O_{(5)\mu}(u)\,\Big\{ x^\mu(2 + \kappa\partial_\kappa) - \tfrac{1}{2}\,\mathrm{i}\kappa\tau\, u^\mu x^2\Big\}\,(3 + \kappa\partial_\kappa)\mathcal{H}_1(\tau u; \kappa x)\,, \tag{13}$$
$$\mathcal{H}_1(\tau u; \kappa x) = \Big(\kappa\tau\sqrt{(ux)^2 - u^2x^2}\Big)^{-1/2} J_{1/2}\Big(\tfrac{\kappa\tau}{2}\sqrt{(ux)^2 - u^2x^2}\Big) e^{\mathrm{i}\kappa\tau(ux)/2}\,,$$

and, instead of performing matrix elements, we continue the investigation of the operator itself. Thereby, as a prerequisite, use is made of a formal Fourier transformation $O_{(5)\alpha}\,(\kappa x, -\kappa x) = \int \mathbf{d}^4 u\, O_{(5)\mu}(u)\, e^{\mathrm{i}\kappa(xu)}$; for simplicity of notation, the measure is written as $\mathbf{d}^4 u = \mathrm{d}^4 u/(2\pi)^4$. The appearance of the non-decomposed operator $\mathcal{O}_\alpha(u)$ looks strange but, in fact, the remaining part of the integrand by construction acts as a projector onto its twist-2 part.

Without further approximations, the operator (12) is used in our geometrically based derivation of generalized WW-relations – as well as other ones – which relate different kinematic structures of that operator only. Let us emphasize again that the notion of geometric twist differs from the dynamical one [9] which is conventionally used in the derivation of generalized WW-relations in DVCS by relating dynamical twist-2 and twist-3 contributions through the equations of motion [26, 27, 28]. (Unfortunately, the different notions of 'twist' – geometrical and dynamical are not the only ones – are often a source of confusion and misunderstanding.)

A general, straightforward procedure of determining the Fourier transform of QCD operators of definite twist, when multiplied by the (leading part of the) propagator, has been introduced in Ref. [29] for scalar operators. In the recent

work [1] it has been applied to the twist-2 (axial) vector operators; these results will be used here. In addition, all computations to be made here, are similar to those which have been done in the case of non-forward scattering and, therefore, will not be repeated in detail; for these explicit computations the reader is referred to [1].

In the following we consider the various irreducible tensor components separately. Thereby, for the clarity of presentation, we use the same ordering of the material within each separate Section, namely, we consider first (A) the structure of the entire Compton operator, then (B) we restrict to the operator expression corresponding to the $s$-channel absorptive part as well as finally (C) we derive the resulting operator relations of the latter.

## III. TRACE PART OF THE TWIST-2 COMPTON OPERATOR

Let us demonstrate the procedure, being sketched in the previous section, for the simplest example – the trace part of the twist-2 Compton operator. Thereby, we also discuss some phenomenological aspects of the various structures.

A) Trace part of the entire operator:
The trace of expression (12) already has been computed in Ref. [1]; it reads

$$\begin{aligned}
\widehat{T}^{\rm tw2}_{\rm trace}(q) &= -2\int \frac{{\rm d}^4x}{2\pi^2}\,\frac{{\rm e}^{{\rm i}qx}\,x^\alpha}{(x^2-{\rm i}\epsilon)^2}\,{\rm i}\Big(O^{\rm tw2}_\alpha(\kappa x,-\kappa x)-O^{\rm tw2}_\alpha(-\kappa x,\kappa x)\Big)\\
&= -2\int_0^1{\rm d}\tau\int \mathbf{d}^4u\,\mathcal{O}_\alpha(u)\,\frac{q^2\,(q^\alpha+\kappa\tau u^\alpha)}{\big[\,(q+\kappa\tau u)^2+{\rm i}\epsilon\big]^2}\,,
\end{aligned}\tag{14}$$

where $\mathcal{O}_\alpha(u)=-\,\mathcal{O}_\alpha(-u)$. Performing the $\tau$–integration partially one obtains

$$\widehat{T}^{\rm tw2}_{\rm trace}(q)=\int\frac{\mathbf{d}^4u}{\kappa^4}\,\mathcal{O}_\alpha\Big(\frac{u}{\kappa}\Big)\,\frac{-q^2}{[(qu)^2-q^2u^2]}\left\{\frac{u^\alpha(q^2+qu)-q^\alpha(qu+u^2)}{R(1)+{\rm i}\epsilon}-\frac{u^\alpha q^2-q^\alpha(qu)}{R(0)+{\rm i}\epsilon}+\int_0^1 d\tau\frac{u^\alpha(qu)-q^\alpha u^2}{R(\tau)+{\rm i}\epsilon}\right\},$$

where $R(\tau)\equiv(q+\tau u)^2$. Introducing the set of antisymmetric operators $\mathcal{O}^{(i)}_\alpha(u)$ of order $i$ as follows,

$$\mathcal{O}^{(0)}_\alpha(u)\equiv\mathcal{O}_\alpha(u)\quad\text{and}\quad\mathcal{O}^{(i)}_\alpha(u)\equiv\int_0^1\frac{d\tau_1}{\tau_1^4}\cdots\int_0^1\frac{d\tau_i}{\tau_i^4}\,\mathcal{O}_\alpha\Big(\frac{u}{\tau_1\ldots\tau_i}\Big)\quad\text{for}\quad i\geq 1,\tag{15}$$

then, after scaling the remaining variables $\tau u$, we obtain

$$\begin{aligned}
\widehat{T}^{\rm tw2}_{\rm trace}(q)=-\int\frac{\mathbf{d}^4u}{\kappa^4}\,\frac{q^2}{[(qu)^2-q^2u^2]}\bigg\{&\big(u^\alpha q^2-q^\alpha(qu)\big)\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big)\left[\frac{1}{R(1)+i\epsilon}-\frac{1}{R(0)+i\epsilon}\right]\\
&+\big(u^\alpha(qu)-q^\alpha u^2\big)\left[\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big)+\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)\right]\frac{1}{R(1)+i\epsilon}\bigg\}.
\end{aligned}\tag{16}$$

This expression has an interesting structure. First, let us remark that in (16) the iterated operators $\mathcal{O}^{(i)}_\alpha$ occur instead of iterated distribution amplitudes which, of course, reappear if appropriate matrix elements are built. (The same occurs later on when the (anti)symmetric part of the entire Compton operator is considered.) Obviously, the appearance of the iterated operators (15) is a consequence of the whole structure of the twist-2 Compton operator. Therefore, it contains essential elements which are typical for the derivation of WW-relations. In addition, the operator combinations of both kinematical structures of (16) are very simply related (if the $1/R(0)$-term is ignored). Let us consider that expression more detailed since it is of technical relevance in our further consideration.

First, introducing $u^\top_\alpha$ as that component of $u_\alpha$ which is transversal with respect to the (external) momentum $q_\alpha$,

$$u^\top_\alpha = g^\top_{\alpha\beta}u^\beta,\qquad g^\top_{\alpha\beta}=g_{\alpha\beta}-q_\alpha q_\beta/q^2\tag{17}$$

$$(u^\top)^2 = [(qu)^2-q^2u^2]\,/\,\big(-q^2\big)\,,\tag{18}$$

we observe that the two independent kinematic structures can be rewritten as follows:

$$v_\alpha \equiv u_\alpha\,q^2-q_\alpha(qu)=q^2\,u^\top_\alpha,\tag{19}$$

$$w_\alpha \equiv u_\alpha(qu)-q_\alpha\,u^2=u^\top_\alpha(qu)-q_\alpha(u^\top)^2.\tag{20}$$

Due to the 'dual' structure of $w$ and $v$ similar relations with $w_\alpha = -u^2 q^\perp_\alpha$ result if we introduce $q^\perp_\alpha = q_\alpha - u_\alpha (qu)/u^2$. In addition, the following relations hold

$$vq = 0 = wu, \qquad wq = -vu = -q^2 (u^\top)^2, \qquad vw = (u^\top)^2 \left[ -q^2/(qu) \right]. \tag{21}$$

At this stage we should mention that the iterated operators $\mathcal{O}^{(i)}_\alpha(u)$ implicitly contain the full kinematical structure of any possible hard QCD process. This becomes obvious when the matrix element of the operator $O^{\rm tw2}_\alpha(x,-x)$ is decomposed into its kinematical independent amplitudes $f_a(x, P_i)$,

$$\langle P_2, S_2 | \, O^{\rm tw2}_\alpha(x,-x) \, | P_1, S_1 \rangle = \textstyle\sum_a \mathcal{K}^a_\alpha(P_i, S_i) \, f_a(x, P_i) \, , \tag{22}$$

where $\mathcal{K}^a_\alpha(P_i, S_i)$ are the independent form factors. An analogous decomposition appears for $\mathcal{O}^{(i)}_\alpha(u)$.

Furthermore, let us decompose $1/R$ into its partial fractions by rewriting

$$\frac{1}{R(\tau) + {\rm i}\epsilon} = \frac{1}{2\sqrt{(qu)^2 - q^2 u^2}} \left( \frac{1}{\tau - \tilde{\xi}_+ + {\rm i}\epsilon} - \frac{1}{\tau - \tilde{\xi}_- - {\rm i}\epsilon} \right), \tag{23}$$

with

$$\tilde{\xi}_\pm = \frac{-\,q^2}{qu \pm \sqrt{(qu)^2 - q^2 u^2}} = \frac{-\,qu \pm \sqrt{(qu)^2 - q^2 u^2}}{u^2} \, . \tag{24}$$

Obviously, these two equivalent forms of $\tilde{\xi}_\pm$ show an analogous structural similarity w.r.t. $u$ and $q$ as has been observed for $v$ and $w$. Let us also remark that due to the antisymmetry of the operators $\mathcal{O}^{(i)}_\alpha(u)$ only the symmetric and antisymmetric combinations of the $1/R$-terms have to be taken when multiplying the kinematic structures $v$ and $w$, respectively.

Now, let us restrict formally the support of $\mathcal{O}(u)$ to regions with $u^2 > 0$ which, in accordance with Ref. [1], makes sense at least if we have in mind $s$-channel processes. Then, we can scale the variable $u$ (having mass dimension 1, $\dim u \equiv [u]$) by the dimensionless 'radius' $t$, $0 \le t \le \infty$, times a unit vector $\hat{p}$ of mass dimension 1,

$$u = t\,\hat{p}\,, \qquad t = \sqrt{u^2/\hat{p}^2}\,, \qquad \hat{p} = \hat{n}\,[u]\,, \qquad \hat{n} = u/\sqrt{u^2}\,, \qquad \hat{n}^2 = 1\,. \tag{25}$$

Then, also $\tilde{\xi}_\pm$ is simply scaled leading to a (dimensionless) analogue $\xi_\pm$ of the Nachtmann variable [4] in terms of a Bjorken-like variable $x$:

$$\tilde{\xi}_\pm = \xi_\pm / t \qquad \text{with} \qquad \xi_\pm = \frac{x}{1 \pm \sqrt{1 + x^2 \hat{p}^2/Q^2}} \qquad \text{and} \qquad x \equiv \frac{Q^2}{q\hat{p}}\,. \tag{26}$$

Furthermore, the integration over $u$ gets an integration over the radius $0 \le t \le \infty$ and over the 4-dimensional unit two-shell hyperboloid $\mathcal{H}^3$.

### B) Restriction to the imaginary part

Next, we restrict to the operator expression ${\rm Im}\,\widehat{T}^{\rm tw2}_{\mu\nu}(q)$ which describes possible absorptive parts in operator form, i.e., that part which results from the imaginary part of the quark propagator. Thereby, the use of the $t$-variable, despite probably not helpful when considering the entire operator (16), is essential for the consideration of its imaginary part. Due to the overall factor $q^2$ in expresseion (16) that imaginary part results from $1/[R(1) + i\epsilon]$ only. Because of Eq. (23), taken at $\tau = 1$, we simply get

$${\rm Im}\,\frac{1}{R(1) + {\rm i}\epsilon} = -\frac{\pi}{2} \frac{1}{\sqrt{(qu)^2 - q^2 u^2}} \left[ \delta(1 - \tilde{\xi}_+) + \delta(1 - \tilde{\xi}_-) \right]. \tag{27}$$

Obviously, it holds

$$\big(qu + u^2\big)\, \delta(1 - \tilde{\xi}_\pm) = \pm \left[ \sqrt{(qu)^2 - q^2 u^2} \right] \delta(1 - \tilde{\xi}_\pm) = -\,\big(q^2 + qu\big)\, \delta(1 - \tilde{\xi}_\pm)\,; \tag{28}$$

these equalities are essential and will be used to simplify some of the resulting expressions below.

In fact, the imaginary part of expression (16) gets

$$\begin{aligned}
\mathrm{Im}\,\widehat{T}^{\pm}_{\rm trace}\,(q) &= \frac{\pi}{2}\int \mathrm{d}t^2 \int \frac{\mathbf{d}^4 u}{\kappa^4}\,\delta(u^2/[u]^2 - t^2)\,\delta\big(1-\tilde{\xi}_\pm\big)\frac{-q^2}{[(qu)^2 - q^2u^2]^{3/2}}\times\\
&\qquad\Big\{\Big[(qu+u^2)\,q^\alpha\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big) + u^2\,q^\alpha\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big] - \Big[(q^2+qu)\,u^\alpha\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big) + (qu)\,u^\alpha\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big]\Big\}\\
&= \pi\int\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\int \mathrm{d}t\,t^4\,\delta\big(t-\xi_\pm\big)\frac{-q^2}{t^3[(q\hat{p})^2 - q^2\hat{p}^2]^{3/2}}\times\\
&\qquad\Big\{\Big[\pm t\,[(q\hat{p})^2 - q^2\hat{p}^2]^{1/2}\,q^\alpha\mathcal{O}^{(0)}_\alpha\Big(\frac{t\hat{p}}{\kappa}\Big) + t^2\,\hat{p}^2\,q^\alpha\mathcal{O}^{(1)}_\alpha\Big(\frac{t\hat{p}}{\kappa}\Big)\Big]\\
&\qquad - \Big[\mp t^2\,[(q\hat{p})^2 - q^2\hat{p}^2]^{1/2}\,\hat{p}^\alpha\mathcal{O}^{(0)}_\alpha\Big(\frac{t\hat{p}}{\kappa}\Big) + t^2\,(q\hat{p})\,\hat{p}^\alpha\mathcal{O}^{(1)}_\alpha\Big(\frac{t\hat{p}}{\kappa}\Big)\Big]\Big\}\\
&= \pi\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\int_0^\infty \mathrm{d}t\,\delta\big(t-\xi_\pm\big)\frac{x}{1+x^2\hat{p}^2/Q^2}\Big\{\pm\Big(1+t\,x\,\frac{\hat{p}^2}{Q^2}\Big)\,\frac{q^\alpha}{q\hat{p}}\,\Omega^{(0)}_\alpha\Big(t,\frac{\hat{p}}{\kappa}\Big)\\
&\qquad + x\,\frac{\hat{p}^2}{Q^2}\Big(\frac{q^\alpha}{q\hat{p}} - \frac{\hat{p}^\alpha}{\hat{p}^2}\Big)\Big[\mp t\,\Omega^{(0)}_\alpha\Big(t,\frac{\hat{p}}{\kappa}\Big) + \frac{1}{\sqrt{1+x^2\hat{p}^2/Q^2}}\,\Omega^{(1)}_\alpha\Big(t,\frac{\hat{p}}{\kappa}\Big)\Big]\Big\}; \qquad (29)
\end{aligned}$$

here and in the following, we maintain $\hat{p}^2 = 1$ in order to remaind its mass dimension without denoting it explicitly. In deriving expression (29), Eq. (28) has been used repeatedly and $t^2\,\mathcal{O}^{(0)}_\alpha(u)$, $t^3\,\mathcal{O}^{(1)}_\alpha(u)$ and so on has been replaced by

$$\Omega^{(0)}_\alpha(t,\hat{p}) \equiv t^2\,\mathcal{O}^{(0)}_\alpha(u)\,,\quad \Omega^{(1)}_\alpha(t,\hat{p}) \equiv t^3\,\mathcal{O}^{(1)}_\alpha(u) = t^3\int_0^1\frac{\mathrm{d}\tau}{\tau^4}\,\mathcal{O}_\alpha\Big(\frac{t\hat{p}}{\tau}\Big) = \int_t^\infty dy\,y^2\,\mathcal{O}_\alpha(y\hat{p}) = \int_t^\infty dy\,\Omega^{(0)}_\alpha(y,\hat{p}), \qquad (30)$$

and so on.

Now, taking into account the following relations

$$\begin{aligned}
\xi_\pm(x,Q^2) &= \frac{x}{1\pm\sqrt{1+x^2\hat{p}^2/Q^2}} = -\frac{Q^2}{\hat{p}^2}\,\frac{1\mp\sqrt{1+x^2\hat{p}^2/Q^2}}{x}, && (31)\\
x\frac{\partial}{\partial x}\Big(\frac{x}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\Big) &= \frac{x}{[1+x^2\hat{p}^2/Q^2]^{3/2}}\,, && (32)\\
x\frac{\partial}{\partial x}\xi_\pm &= \pm\frac{\xi_\pm}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\,, && (33)\\
x\frac{\partial}{\partial x}\Omega^{(1)}_\alpha(\xi_\pm,\hat{p}) &= \mp\frac{\xi_\pm}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\Omega^{(0)}_\alpha(\xi_\pm,\hat{p})\,, && (34)
\end{aligned}$$

and introducing the following operator functions,

$$\begin{aligned}
\mathcal{V}_{0\alpha}(\xi_\pm,x;\hat{p}) &\equiv \frac{x\,\Omega^{(0)}_\alpha(\xi_\pm,\hat{p})}{\sqrt{1+x^2\hat{p}^2/Q^2}}\,, && (35)\\
\mathcal{V}_{1\alpha}(\xi_\pm,x;\hat{p}) &\equiv x\frac{\partial}{\partial x}\Bigg(\frac{x\,\Omega^{(1)}_\alpha(\xi_\pm,\hat{p})}{\sqrt{1+x^2\hat{p}^2/Q^2}}\Bigg) = \frac{x}{1+x^2\hat{p}^2/Q^2}\Bigg[\mp\,\xi_\pm\,\Omega^{(0)}_\alpha(\xi_\pm,\hat{p}) + \frac{1}{\sqrt{1+x^2\hat{p}^2/Q^2}}\,\Omega^{(1)}_\alpha(\xi_\pm,\hat{p})\Bigg], && (36)
\end{aligned}$$

one finally finds for the imaginary part of the trace of the twist-2 Compton operator:

$$\begin{aligned}
\mathrm{Im}\,\widehat{T}^{\rm tw2}_{\rm trace}\,(q) &= 2\pi\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\Big\{\frac{q_\alpha}{q\hat{p}}\,\mathcal{V}^\alpha_0\Big(x,\frac{\hat{p}}{\kappa}\Big) + x\,\frac{\hat{p}^2}{Q^2}\Big(\frac{q_\alpha}{q\hat{p}} - \frac{\hat{p}_\alpha}{\hat{p}^2}\Big)\,\mathcal{V}^\alpha_1\Big(x,\frac{\hat{p}}{\kappa}\Big)\Big\} && (37)\\
&= 2\pi\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\Big\{\frac{q_\alpha}{q\hat{p}}\Big[\mathcal{V}^\alpha_0\Big(x,\frac{\hat{p}}{\kappa}\Big) + \frac{\big(\hat{p}^\top\big)^2}{q\hat{p}}\mathcal{V}^\alpha_1\Big(x,\frac{\hat{p}}{\kappa}\Big)\Big] - \frac{\hat{p}^\top_\alpha}{q\hat{p}}\,\mathcal{V}^\alpha_1\Big(x,\frac{\hat{p}}{\kappa}\Big)\Big\} && (38)
\end{aligned}$$

with $\mathcal{V}_{i\alpha}(x,\hat{p}) = \mathcal{V}_{i\alpha}(\xi_+,x;\hat{p}) + \mathcal{V}_{i\alpha}(\xi_-,x;\hat{p})$, $i = 0, 1$.

## IV. ANTISYMMETRIC PART OF THE TWIST-2 COMPTON OPERATOR

### A) Entire operator expression

Now, we consider the antisymmetric part of the Compton operator which is given by [1]

$$\begin{aligned}\widehat{T}^{\rm tw2}_{[\mu\nu]}(q) &= \epsilon_{\mu\nu}{}^{\alpha\beta}\int\frac{{\rm d}^4x}{2\pi^2}\,\frac{{\rm e}^{iqx}\,x_\alpha}{(x^2-{\rm i}\epsilon)^2}\,\Big(O^{\rm tw2}_{5\beta}(\kappa x,-\kappa x)+O^{\rm tw2}_{5\beta}(-\kappa x,\kappa x)\Big)\\ &= \epsilon_{\mu\nu}{}^{\alpha\beta}\int_0^1{\rm d}\tau\int_0^1{\rm d}\tau'\int\frac{\mathbf{d}^4u}{\kappa^4}\,\mathcal{O}_{5\rho}\left(\frac{u}{\kappa}\right)\,\partial^\rho_u\left(\frac{q^2\,q_{[\alpha}\,u_{\beta]}}{[(q+\tau\tau' u)^2+i\epsilon]^2}\right).\end{aligned} \tag{39}$$

Again, the $\tau$–integrations may be performed partially so that we end up with $1/[R(1)+{\rm i}\epsilon]$ as well as single and double integrals over $1/[R(\tau)+{\rm i}\epsilon]$ and $1/[R(\tau\tau')+{\rm i}\epsilon]$, respectively. Then, performing that tedious but straightforward calculation and scaling away the respective $\tau-$dependence, the result finally reads (cf. computations in Ref. [1])

$$\begin{aligned}\widehat{T}^{\rm tw2}_{[\mu\nu]}(q) = \int\frac{\mathbf{d}^4u}{\kappa^4}\,\frac{-q^2\,\epsilon_{\mu\nu}{}^{\alpha\beta}\,q_\alpha}{2\,[(qu)^2-q^2u^2]}\,\frac{1}{R(1)+{\rm i}\epsilon}\,&\Bigg\{g^{\top\rho}_\beta\Big[(qu+u^2)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+u^2\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\Big]\\ &+u^\top_\beta\,q^\rho\left[\mathcal{O}^{(0)}_{5\rho}\left(\frac{u}{\kappa}\right)+2\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)-3\frac{qu}{[(qu)^2-q^2u^2]}\left[(qu+u^2)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+u^2\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\right]\right]\\ &+u^\top_\beta\,u^\rho\left[\mathcal{O}^{(0)}_{5\rho}\left(\frac{u}{\kappa}\right)-\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)+3\frac{qu}{[(qu)^2-q^2u^2]}\left[(q^2+qu)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+(qu)\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\right]\right]\Bigg\},\end{aligned} \tag{40}$$

where the set of operators $\mathcal{O}^{(i)}_{5\mu}(u)$ are defined in the same manner as $\mathcal{O}^{(i)}_{\mu}(u)$ in Eqs. (15); possible terms proportional to $1/[R(0)+{\rm i}\epsilon]$ vanish due to the symmetry of the operators, $\mathcal{O}^{(i)}_{5\alpha}(u)=\mathcal{O}^{(i)}_{5\alpha}(-u)$. The transverse structures are due to the $\epsilon$-tensor. With the help of $v_\rho=q^2u^\top_\rho$ and $w_\rho=(qu)u^\top_\rho-(u^\top_\rho)^2q_\rho$ we get another structure,

$$\begin{aligned}\widehat{T}^{\rm tw2}_{[\mu\nu]}(q) = \int\frac{\mathbf{d}^4u}{\kappa^4}\,\frac{-q^2\,\epsilon_{\mu\nu}{}^{\alpha\beta}\,q_\alpha}{2\,[(qu)^2-q^2u^2]}\,\frac{1}{R(1)+{\rm i}\epsilon}\,&\Bigg\{g^{\top\rho}_\beta\Big[(qu+u^2)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+u^2\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\Big]\\ &+\frac{u^\top_\beta\,w^\rho}{(qu)^2-q^2u^2}\left[\left((q^2+qu)\,\mathcal{O}^{(0)}_{5\rho}\left(\frac{u}{\kappa}\right)+(qu)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)\right)+2\left((q^2+qu)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+(qu)\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\right)\right]\\ &-\frac{u^\top_\beta\,v^\rho}{(qu)^2-q^2u^2}\left[\left((qu+u^2)\,\mathcal{O}^{(0)}_{5\rho}\left(\frac{u}{\kappa}\right)+u^2\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)\right)-\left((qu+u^2)\,\mathcal{O}^{(1)}_{5\rho}\left(\frac{u}{\kappa}\right)+u^2\,\mathcal{O}^{(2)}_{5\rho}\left(\frac{u}{\kappa}\right)\right)\right]\Bigg\},\end{aligned} \tag{41}$$

where the kinematics is now classified according to $g^{\top\rho}_\beta\mathcal{O}^{(i)}_{5\rho},\ v^\rho\mathcal{O}^{(i)}_{5\rho}$ and $w^\rho\mathcal{O}^{(i)}_{5\rho}$. Especially in the latter case one observes remarkable structural similarities of these various expressions in terms of the iterated operators. However, relations being similar to those which will be derived now for the imaginary part have not been found.

### B) Restriction to the imaginary part ${\rm Im}\,\widehat{T}^{\rm tw2}_{[\mu\nu]}(q)$

Now, let us use in Eq. (40) the representation (27) of the imaginary part of $1/[R(1)+{\rm i}\epsilon]$ and performing the same steps as in Eq. (29). Namely, introducing the additional $t-$integration, rewriting $u=t\hat p$ and replacing the set of operators $t^3\mathcal{O}^{(i)}_\alpha(t\,\hat p)$ by

$$\Omega^{(0)}_{5\alpha}(t,\hat p)=t^3\,\mathcal{O}_\alpha(t\,\hat p)\,,\qquad \Omega^{(i)}_{5\alpha}(t,\hat p)=\int_t^\infty\frac{{\rm d}y}{y}\,\Omega^{(i-1)}_{5\alpha}(y,\,\hat p)\qquad\text{for}\qquad i\geq 1\,, \tag{42}$$

we obtain

$$\begin{aligned}{\rm Im}\,\widehat{T}^{\rm tw2}_{[\mu\nu]}(q) &= {\rm Im}\,\widehat{T}^{+}_{[\mu\nu]}(q)+{\rm Im}\,\widehat{T}^{-}_{[\mu\nu]}(q)\,,\\ {\rm Im}\,\widehat{T}^{\pm}_{[\mu\nu]}(q) &= \frac{\pi}{2}\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat p}{\kappa^4}\,\delta(\hat p^2-1)\,\epsilon_{\mu\nu}{}^{\alpha\beta}\left(-\frac{x}{\xi_\pm}\,\frac{1}{[1+x^2\hat p^2/Q^2]^{3/2}}\right)\times\\ &\quad\Bigg\{\frac{q_\alpha g^\rho_\beta}{q\hat p}\left[\left(1+x\,\xi_\pm\frac{\hat p^2}{Q^2}\right)\Omega^{(1)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)+x\,\xi_\pm\,\frac{\hat p^2}{Q^2}\,\Omega^{(2)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)\right]\\ &\quad+\frac{q_\alpha\,\hat p_\beta}{q\hat p}\frac{q^\rho}{q\hat p}\left[\Omega^{(0)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)\mp\frac{1-2\,x\,\xi_\pm\,\hat p^2/Q^2}{[1+x^2\hat p^2/Q^2]^{1/2}}\,\Omega^{(1)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)-\frac{3\,x\,\xi_\pm\,\hat p^2/Q^2}{[1+x^2\hat p^2/Q^2]}\,\Omega^{(2)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)\right]\\ &\quad+x\,\xi_\pm\,\frac{\hat p^2}{Q^2}\,\frac{q_\alpha\,\hat p_\beta}{q\hat p}\,\frac{\hat p^\rho}{\hat p^2}\left[\Omega^{(0)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)\mp\frac{3}{[1+x^2\hat p^2/Q^2]^{1/2}}\,\Omega^{(1)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)+\frac{2-x^2\hat p^2/Q^2}{[1+x^2\hat p^2/Q^2]}\,\Omega^{(2)}_{5\rho}\left(\xi_\pm,\frac{\hat p}{\kappa}\right)\right]\Bigg\}.\end{aligned} \tag{43}$$

Now, observing Eqs. (33) and (34) together with

$$x\frac{\partial}{\partial x}\left(\frac{x}{\xi_\pm}\frac{1}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\right) \;=\; \frac{x}{\xi_\pm}\frac{1\mp[1+x^2\hat{p}^2/Q^2]^{1/2}}{[1+x^2\hat{p}^2/Q^2]^{3/2}} = -\frac{x^2\hat{p}^2/Q^2}{[1+x^2\hat{p}^2/Q^2]^{3/2}}\,,$$

we get

$$\begin{aligned}\mathrm{Im}\,\widehat{T}^{\mathrm{tw2}}_{[\mu\nu]}(q) = -\,\frac{\pi}{2}\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\,\epsilon_{\mu\nu}{}^{\alpha\beta}\Bigg\{&\frac{q_\alpha\, g_\beta{}^\rho}{q\hat{p}}\left[\mathcal{G}_{1\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)+\mathcal{G}_{2\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)\right]\\ &-\frac{q_\alpha\,\hat{p}_\beta}{q\hat{p}}\left[\frac{q^\rho}{q\hat{p}}\,\mathcal{G}_{2\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)-\frac{\hat{p}^\rho}{Q^2}\,\mathcal{G}_{0\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)\right]\Bigg\}\,,\end{aligned} \tag{44}$$

with $\mathcal{G}_{i\rho}(x,\hat{p}/\kappa)=\mathcal{G}^{+}_{i\rho}(x,\hat{p}/\kappa)+\mathcal{G}^{-}_{i\rho}(x,\hat{p}/\kappa)$ for $i=0,1,2$, and

$$\mathcal{G}^{\pm}_{1\rho}(x,\hat{p}/\kappa) \equiv x\frac{\partial}{\partial x}\left[x\frac{\partial}{\partial x}\left(\frac{x}{\xi_\pm}\frac{\Omega^{(2)}_{5\rho}(\xi_\pm,\hat{p}/\kappa)}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\right)\right], \tag{45}$$

$$\mathcal{G}^{\pm}_{2\rho}(x,\hat{p}/\kappa) \equiv -x\frac{\partial^2}{\partial x^2}x\left(\frac{x}{\xi_\pm}\frac{\Omega^{(2)}_{5\rho}(\xi_\pm,\hat{p}/\kappa)}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\right), \tag{46}$$

$$\mathcal{G}^{\pm}_{0\rho}(x,\hat{p}/\kappa) \equiv x^2\,\frac{\partial^2}{\partial x^2}\left(x^2\frac{\Omega^{(2)}_{5\rho}(\xi_\pm,\hat{p}/\kappa)}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\right). \tag{47}$$

C) General operator relations between $\mathcal{G}^{\pm}_{i\,\rho}(x,\hat{p}/\kappa)$

Obviously, Eqs. (45) – (47) contain only two independent quantities which contribute, namely, introducing $\mathcal{F}_\rho(x,\hat{p}/\kappa)=\mathcal{F}^{+}_\rho(x,\hat{p}/\kappa)+\mathcal{F}^{-}_\rho(x,\hat{p}/\kappa)$ and $\mathcal{F}^{0}_\rho(x,\hat{p}/\kappa)=\xi_+\mathcal{F}^{+}_\rho(x,\hat{p}/\kappa)+\xi_-\mathcal{F}^{-}_\rho(x,\hat{p}/\kappa)$ with

$$\mathcal{F}^{\pm}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)=\frac{x}{\xi_\pm}\frac{\Omega^{(2)}_{5\rho}(\xi_\pm,\hat{p}/\kappa)}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\,, \tag{48}$$

we may rewrite the relations (45) – (47) as follows:

$$\begin{aligned}\mathcal{G}_{1\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) &= x\frac{\partial}{\partial x}\,x\frac{\partial}{\partial x}\,\mathcal{F}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)\,,\\ \mathcal{G}_{2\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) &= -x\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}+1\right)\mathcal{F}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)\,,\\ \mathcal{G}_{0\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) &= x\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}-1\right)x\,\mathcal{F}^{0}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)\,.\end{aligned} \tag{49}$$

The first two of these relations are formally identical in their functional dependence with the corresponding equations in DIS as given in Ref. [30]. Therefore, they will have the same general consequences: Observing

$$x\frac{\partial}{\partial x}\mathcal{F}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)=-\int_x^1\frac{dy}{y}\mathcal{G}_{1\rho}\Big(y,\frac{\hat{p}}{\kappa}\Big)\,,\qquad \mathcal{F}_\rho\Big(x,\frac{\hat{p}}{\kappa}\Big)=-\int_x^1\frac{dy}{y}\int_y^1\frac{dy'}{y'}\mathcal{G}_{1\rho}\Big(y',\frac{\hat{p}}{\kappa}\Big)\,,$$

we find that, surprisingly, the well known Wandzura-Wilczek relation from deep inelastic $e\,p-$scattering [3] have an operator analogue, namely, the (twist-2 part of) $\mathcal{G}_{2\rho}(x,\hat{p}/\kappa)$ fulfills the same relation as does the (twist-2 part of the) structure function $g_2(x)$ of DIS,

$$\mathcal{G}_{2\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)\equiv\mathcal{G}^{\mathrm{WW}}_{2\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) \;=\; -\,\mathcal{G}_{1\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big)+\int_x^1\frac{dy}{y}\mathcal{G}_{1\rho}\Big(y,\frac{\hat{p}}{\kappa}\Big)\,. \tag{50}$$

This means that we are confronted with a very general structure of the theory which is independent of taking matrix elements. Instead, it is a property of the leading twist-2 part of the Fourier transformed Compton operator.

In addition, another relation holds for the (twist-2 part of the) operator expression $\mathcal{G}_{0\rho}(x,\hat{p}/\kappa)$ which is unknown in the structure of DIS and is suppressed by a factor $1/Q^2$:

$$\mathcal{G}^{\pm}_{0\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) = (x\xi_\pm)\,\mathcal{G}^{\pm}_{1\rho}\Big(x,\frac{\hat{p}}{\kappa}\Big) - \frac{2x^2+x\xi_\pm}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\int_x^1\frac{dy}{y}\mathcal{G}^{\pm}_{1\rho}\Big(y,\frac{\hat{p}}{\kappa}\Big) - \frac{2x^2}{[1+x^2\hat{p}^2/Q^2]^{3/2}}\int_x^1\frac{dy}{y}\int_y^1\frac{dy'}{y'}\mathcal{G}^{\pm}_{1\rho}\Big(y',\frac{\hat{p}}{\kappa}\Big)\,. \tag{51}$$

It is remarkable that only one kind of operators, namely $\mathcal{G}^{\pm}_{1\rho}(x,\hat{p}/\kappa)$ or, equivalently, $\Omega^{(2)}_{5\rho}(\xi_\pm,\hat{p}/\kappa)$, determines the complete structure of the antisymmetric piece of the (imaginary part of the) Compton operator. Let us emphasize that this obtains without using any dynamical assumption but, due to the twist-2 structure of the operator, is a purely geometric result. Furthermore, these relations respect the complete power or, after taking matrix elements, target mass corrections which result from the (infinite number of) trace terms of the twist-2 Compton operator.

## V. SYMMETRIC PART OF THE TWIST-2 COMPTON OPERATOR

### A) Entire operator expression

Now we consider the symmetric part of the Compton operator which is given by [1]

$$\begin{aligned}
\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q) &= {\rm i}\,S^{\mu\nu|\alpha\beta}\int\frac{\mathbf{d}^4x}{2\pi^2}\,\frac{{\rm e}^{{\rm i}qx}\,x_\alpha}{(x^2-{\rm i}\epsilon)^2}\,\big(O^{\rm tw2}_\beta(\kappa x,-\kappa x)-O^{\rm tw2}_\beta(-\kappa x,\kappa x)\big)\\
&= 2\int_0^1\frac{{\rm d}\tau}{\tau}\,(1-\tau+\tau\ln\tau)\int\frac{\mathbf{d}^4u}{\kappa^4\tau^4}\,\mathcal{O}_\rho\Big(\frac{u}{\kappa\tau}\Big)\,\partial^\rho_u\Bigg[\frac{2}{[(q+u)^2+i\epsilon]^3}\times\\
&\qquad\Big\{\big(q^2\,g_{\mu\nu}-q_\mu q_\nu\big)\,\big[(uq)^2-u^2q^2+\tfrac12 u^2(q+u)^2\big]+\big(u_\mu q^2-q_\mu\,(uq)\big)\big(u_\nu q^2-q_\nu\,(uq)\big)\Big\}\Bigg]\\
&= 2\int_0^1 d\tau\int_0^1 d\sigma\int_0^1 d\rho\int\frac{\mathbf{d}^4u}{\kappa^4}\,\mathcal{O}_\rho\Big(\frac{u}{\kappa}\Big)\,\big[q^2\big]^3\,\partial^\rho_{\widetilde{u}}\Bigg\{\frac{2A^\top_{\mu\nu}(q,\widetilde{u})}{\big[\widetilde{R}(\tau)+{\rm i}\epsilon\big]^3}+\frac{B^\top_{\mu\nu}(q,\widetilde{u})}{\big[\widetilde{R}(\tau)+{\rm i}\epsilon\big]^2}\Bigg\},
\end{aligned}\tag{52}$$

thereby, we introduced the following abbreviations:

$$A^\top_{\mu\nu}(q,u) = g^\top_{\mu\nu}\Big(\frac{qu}{q^2}\Big)^2\Big(1-\frac{q^2u^2}{(qu)^2}\Big)+\frac{1}{q^2}u^\top_\mu u^\top_\nu\,,\qquad B^\top_{\mu\nu}(q,u) = g^\top_{\mu\nu}\frac{u^2}{[q^2]^2}\,,\tag{53}$$

$$\widetilde{R}(\tau) = (q+\tau\widetilde{u})^2\,,\qquad \widetilde{u}_\mu = \sigma\rho\,u_\mu\,.\tag{54}$$

Again, we have to compute the various integrations over $\rho$, $\sigma$ and $\tau$ partially, together with the derivation w.r.t. $u$, so that finally integrals over $1/[R+{\rm i}\epsilon]$ only remain. Then, rescaling and introducing antisymmetric operators $\mathcal{O}^{(i)}_\alpha(u)$ according to Eq. (15) the result of the tedious but straightforward calculation is as follows (cf. also Ref. [1]):

$$\begin{aligned}
\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q) &= \frac{q^2}{2}\int\frac{\mathbf{d}^4u}{\kappa^4}\Bigg\{\frac{q_\alpha}{qu}\Big[g^\top_{\mu\nu}\,\mathcal{F}^\alpha_1\Big(\frac{u}{\kappa}\Big)+\frac{q^2u^\top_\mu u^\top_\nu}{(qu)^2-q^2u^2}\,\mathcal{F}^\alpha_2\Big(\frac{u}{\kappa}\Big)\Big]\\
&\qquad+\Big(\frac{q_\alpha}{qu}-\frac{u_\alpha}{u^2}\Big)\Big[g^\top_{\mu\nu}\,\mathcal{F}^\alpha_3\Big(\frac{u}{\kappa}\Big)+\frac{q^2u^\top_\mu u^\top_\nu}{(qu)^2-q^2u^2}\,\mathcal{F}^\alpha_4\Big(\frac{u}{\kappa}\Big)\Big]\\
&\qquad+\Big(u^\top_\mu g^\top_{\nu\alpha}+u^\top_\nu g^\top_{\mu\alpha}-2\,u^\top_\mu u^\top_\nu\frac{q_\alpha}{qu}\Big)\frac{q^2}{(qu)^2-q^2u^2}\,\mathcal{F}^\alpha_5\Big(\frac{u}{\kappa}\Big)\Bigg\}\frac{1}{R(1)+{\rm i}\epsilon}
\end{aligned}\tag{55}$$

where $\mathcal{F}_{i\alpha}\,(u/\kappa)$ are abbreviations of the following combinations:

$$\mathcal{F}_{1\alpha}\Big(\frac{u}{\kappa}\Big) = \mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big)+\frac{u^2}{(qu)^2-q^2u^2}\,\Big((qu+u^2)\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)+u^2\,\mathcal{O}^{(2)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big),\tag{56}$$

$$\mathcal{F}_{2\alpha}\Big(\frac{u}{\kappa}\Big) = \mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big)+\frac{3\,u^2}{(qu)^2-q^2u^2}\,\Big((qu+u^2)\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)+u^2\,\mathcal{O}^{(2)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big),\tag{57}$$

$$\mathcal{F}_{3\alpha}\left(\frac{u}{\kappa}\right) = -\int_0^1 \frac{d\tau}{\tau^2}\left[\mathcal{F}_{1\alpha}\left(\frac{u}{\kappa\tau}\right) + \frac{(qu)^2}{(qu)^2-q^2u^2}\,\mathcal{F}_{2\alpha}\left(\frac{u}{\kappa\tau}\right)\right] - \frac{u^2(q+u)^2}{(qu)^2-q^2u^2}\int_0^1 \frac{d\tau}{\tau^2}\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa\tau}\Big)$$
$$+\frac{2(qu)}{(qu)^2-q^2u^2}\int_0^1 \frac{d\tau}{\tau^2}\Big((qu+u^2)\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa\tau}\Big) + u^2\,\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa\tau}\Big)\Big)$$
$$+\frac{u^2}{(qu)^2-q^2u^2}\Big((q^2+qu)\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big) + qu\,\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big)\,, \tag{58}$$

$$\mathcal{F}_{4\alpha}\left(\frac{u}{\kappa}\right) = 3\,\mathcal{F}_{3\alpha}\left(\frac{u}{\kappa}\right) - \frac{2\,q^2u^2}{(qu)^2-q^2u^2}\,\mathcal{F}_{5\alpha}\left(\frac{u}{\kappa}\right) - \frac{2\,u^2}{(qu)^2-q^2u^2}\Big((q^2+qu)\,\mathcal{O}^{(0)}_\alpha\Big(\frac{u}{\kappa}\Big) + qu\,\mathcal{O}^{(1)}_\alpha\Big(\frac{u}{\kappa}\Big)\Big)\,, \tag{59}$$

$$\mathcal{F}_{5\alpha}\left(\frac{u}{\kappa}\right) = \int_0^1 \frac{d\tau}{\tau^2}\,\mathcal{F}_{2\alpha}\left(\frac{u}{\kappa\tau}\right)\,. \tag{60}$$

First, we observe the remarkable structural similarity of $\mathcal{F}_{1\alpha}$ and $\mathcal{F}_{2\alpha}$; it is the source of the Callan-Gross relation which will be shown below. Second, we observe that $\mathcal{F}_{3\alpha}$ and $\mathcal{F}_{4\alpha}$ are determined by $\mathcal{F}_{1\alpha}$ and $\mathcal{F}_{2\alpha}$ and furthermore contain also those operator combinations of $\mathcal{O}^{(0)}_\alpha$ and $\mathcal{O}^{(1)}_\alpha$ which already appeared in the trace part, Eq. (29). Finally, we remark that only $\mathcal{F}_{1\alpha}$ and $\mathcal{F}_{2\alpha}$ survive when forward matrix elements are taken.

Taking the trace of expression (55) one gets

$$\widehat{T}^{\rm tw2}_{\rm trace}\,(q) \;=\; \frac{q^2}{2}\int\frac{\mathbf{d}^4u}{\kappa^4}\,\Big\{\frac{q_\alpha}{qu}\Big[\,3\,\mathcal{F}^\alpha_1\left(\frac{u}{\kappa}\right) - \mathcal{F}^\alpha_2\left(\frac{u}{\kappa}\right)\Big]$$
$$+\Big(\frac{q_\alpha}{qu}-\frac{u_\alpha}{u^2}\Big)\Big[\,3\,\mathcal{F}^\alpha_3\left(\frac{u}{\kappa}\right) - \mathcal{F}^\alpha_4\left(\frac{u}{\kappa}\right) - \frac{2\,q^2u^2}{(qu)^2-q^2u^2}\,\mathcal{F}^\alpha_5\left(\frac{u}{\kappa}\right)\Big]\Big\}\frac{1}{R(1)+\mathrm{i}\epsilon}\,; \tag{61}$$

as one convinces oneself, it coincides, modulo $1/R(0)$-terms, with the former result (16) which we determined independently by another way.

From this for the symmetric *traceless* Compton operator we get

$$\widehat{T}^{\rm tw2}_{\{\mu\nu\}\rm traceless}\,(q) \;=\; \widehat{T}^{\rm tw2}_{\{\mu\nu\}}\,(q) - \frac{1}{3}\,g^\top_{\mu\nu}\,\widehat{T}^{\rm tw2}_{\rm trace}\,(q)$$
$$=\;\frac{q^2}{2}\int\frac{\mathbf{d}^4u}{\kappa^4}\,\Big\{\Big(\frac{1}{3}g^\top_{\mu\nu} - \frac{u^\top_\mu u^\top_\nu}{(u^\top)^2}\Big)\Big[\,\frac{q_\alpha}{qu}\Big(\mathcal{F}^\alpha_2\left(\frac{u}{\kappa}\right) + \frac{(u^\top)^2}{u^2}\mathcal{F}^\alpha_4\left(\frac{u}{\kappa}\right) - 2\mathcal{F}^\alpha_5\left(\frac{u}{\kappa}\right)\Big) - \frac{u^\top_\alpha}{u^2}\,\mathcal{F}^\alpha_4\left(\frac{u}{\kappa}\right)\Big]$$
$$+\Big(\frac{2\,g^\top_{\mu\nu}u^\top_\alpha}{3(u^\top)^2} - \frac{u^\top_\mu g^\top_{\nu\alpha}+u^\top_\nu g^\top_{\mu\alpha}}{(u^\top)^2}\Big)\mathcal{F}^\alpha_5\left(\frac{u}{\kappa}\right)\Big\}\frac{1}{R(1)+\mathrm{i}\epsilon}. \tag{62}$$

Obviously, $\mathcal{F}^\alpha_1$ and $\mathcal{F}^\alpha_3$ disappeared since they are trace parts. Expressions (55), (61) and (62) correspond to a kinematic expansion of related amplitudes.

### B) Restriction to the imaginary part $\mathrm{Im}\,\widehat{T}^{\rm tw2}_{\{\mu\nu\}}\,(q)$

Now, let us use in Eq. (55) the representation (27) of the imaginary part of $1/[R(1)+\mathrm{i}\epsilon]$ and, again, perform the same steps as in Eq. (29).

#### *1. Callan-Gross operator relation:* $\mathcal{F}^\alpha_1$ *and* $\mathcal{F}^\alpha_2$

First, we consider that contribution to the Compton operator which remains if forward matrix elements are taken, i.e., which is given by the combinations $\mathcal{F}^\alpha_1$ and $\mathcal{F}^\alpha_2$ only; it will be denoted by ${}^0\widehat{T}_{\{\mu\nu\}}\,(q)$. Let us take the imaginary part of $1/[R(1)+\mathrm{i}\epsilon]$ and let us scale the $u-$variable as previously has been done,

$$\mathrm{Im}\,{}^0\widehat{T}^{\rm tw2}_{\{\mu\nu\}}\,(q) = \mathrm{Im}\,{}^0\widehat{T}^{+}_{\{\mu\nu\}}\,(q) + \mathrm{Im}\,{}^0\widehat{T}^{-}_{\{\mu\nu\}}\,(q)\,,$$
$$\mathrm{Im}\,{}^0\widehat{T}^{\pm}_{\{\mu\nu\}}\,(q) = \frac{-\pi}{2}\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat p}{\kappa^4}\,\delta(\hat p^2-1)\int\mathrm{d}t\,t^2\,\delta\big(t-\tilde\xi_\pm\big)\frac{q^\alpha}{q\hat p}$$
$$\times\;\Big[\frac{q^2 g^\top_{\mu\nu}}{[(q\hat p)^2-q^2\hat p^2]^{1/2}}\Big\{\mathcal{O}^{(0)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big) \pm \frac{t\,\hat p^2}{[(q\hat p)^2-q^2\hat p^2]^{1/2}}\mathcal{O}^{(1)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big) + \frac{t^2\,(\hat p^2)^2}{[(q\hat p)^2-q^2\hat p^2]}\mathcal{O}^{(2)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big)\Big\}$$
$$+\;\frac{[q^2]^2\,\hat p^\top_\mu\hat p^\top_\nu}{[(q\hat p)^2-q^2\hat p^2]^{3/2}}\Big\{\mathcal{O}^{(0)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big) \pm \frac{3\,t\,\hat p^2}{[(q\hat p)^2-q^2\hat p^2]^{1/2}}\mathcal{O}^{(1)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big) + \frac{3\,t^2\,(\hat p^2)^2}{[(q\hat p)^2-q^2\hat p^2]}\mathcal{O}^{(2)}_\alpha\Big(\frac{t\hat p}{\kappa}\Big)\Big\}\Big]$$

$$
\begin{aligned}
&= \frac{\pi}{2}\int_{\mathcal{H}^3} \frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\frac{q^\alpha}{q\hat{p}} \\
&\times\Bigg[\frac{x g^{\top}_{\mu\nu}}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\Big\{\Omega^{(0)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big) \pm \frac{x\hat{p}^2/Q^2}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\Omega^{(1)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big) + \frac{x^2(\hat{p}^2/Q^2)^2}{[1+x^2\hat{p}^2/Q^2]}\Omega^{(2)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big)\Big\} \\
&\quad - \frac{x^3\hat{p}^{\top}_\mu\hat{p}^{\top}_\nu/Q^2}{[1+x^2\hat{p}^2/Q^2]^{3/2}}\Big\{\Omega^{(0)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big) \pm \frac{3\,x\hat{p}^2/Q^2}{[1+x^2\hat{p}^2/Q^2]^{1/2}}\Omega^{(1)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big) + \frac{3\,x^2(\hat{p}^2/Q^2)^2}{[1+x^2\hat{p}^2/Q^2]}\Omega^{(2)}_\alpha\Big(\xi_\pm,\frac{\hat{p}}{\kappa}\Big)\Big\}\Bigg]
\end{aligned}
\tag{63}
$$

where we used the distribution amplitudes (30) and extended them to arbitrary order $i$:

$$
\Omega^{(0)}_\alpha(t,\hat{p}) \equiv t^2\,\mathcal{O}^{(0)}_\alpha(t\,\hat{p})\;, \qquad \Omega^{(i)}_a(t,\hat{p}) \equiv \int_t^1 dy\,\Omega^{(i-1)}_\alpha(y,\hat{p}) \quad \text{for} \quad i\geq 1\,.
$$

Now, let us look for possible relations between the kinematical structures of expression (63).

Formally, the structure in the angular bracket of expression (63) is completely analogous to the corresponding one of the structure functions $W_1$ and $W_2$ in DIS as given by Georgi and Politzer [6]. Therefore, let us introduce the operators $\mathcal{W}_1$ and $\mathcal{W}_2$ according to

$$
\mathrm{Im}\,{}^0\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q) \;=\; \pi\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\frac{q^\alpha}{q\hat{p}}\Big[-g^{\top}_{\mu\nu}\,\mathcal{W}_{1\alpha}\Big(x,\frac{\hat{p}}{\kappa}\Big) + \frac{\hat{p}^{\top}_\mu\hat{p}^{\top}_\nu}{\hat{p}^2}\,\mathcal{W}_{2\alpha}\Big(x,\frac{\hat{p}}{\kappa}\Big)\Big]\,. \tag{64}
$$

Then, we observe that both operators may be written as:

$$
\begin{aligned}
\mathcal{W}_{i\alpha}\Big(x,\frac{\hat{p}}{\kappa}\Big) &= \mathcal{W}_{i\alpha}\Big(\xi_+,x;\frac{\hat{p}}{\kappa}\Big) + \mathcal{W}_{i\alpha}\Big(\xi_-,x;\frac{\hat{p}}{\kappa}\Big) \qquad \text{for} \qquad i=1,2,L\,, \\
x\,\mathcal{W}_{1\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) &= \frac{(q\hat{p})}{\hat{p}^2}\,\mathcal{W}_{2\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) + x^2\,\frac{\hat{p}^2}{Q^2}\Big[\Big(x\frac{\partial}{\partial x} + x^2\frac{\partial^2}{\partial x^2}\,\frac{x}{\xi_\pm}\Big)\Big(\frac{x}{\xi_\pm}\,\frac{\Omega^{(2)}_\alpha(\xi_\pm,\hat{p}/\kappa)}{\sqrt{1+x^2\hat{p}^2/Q^2}}\Big)\Big] \\
&= (1+x^2\hat{p}^2/Q^2)\,\frac{(q\hat{p})}{\hat{p}^2}\,\mathcal{W}_{2\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) - x\,\mathcal{W}_{L\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big)\,,
\end{aligned}
\tag{65}
$$

$$
\frac{(q\hat{p})}{\hat{p}^2}\,\mathcal{W}_{2\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) \;=\; x^2\frac{\partial^2}{\partial x^2}\Big(\frac{x^2}{\xi_\pm^2}\,\frac{\Omega^{(2)}_\alpha(\xi_\pm,\hat{p}/\kappa)}{\sqrt{1+x^2\hat{p}^2/Q^2}}\Big)\,, \tag{66}
$$

$$
x\,\mathcal{W}_{L\alpha}\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) \;=\; -x^2\,\frac{\hat{p}^2}{Q^2}\,x\frac{\partial}{\partial x}\Big(\frac{x}{\xi_\pm}\,\frac{\Omega^{(2)}_\alpha(\xi_\pm,\hat{p}/\kappa)}{\sqrt{1+x^2\hat{p}^2/Q^2}}\Big)\,. \tag{67}
$$

Obviously, equation (65) is the operator version of the well-known (target mass corrected) Callan-Gross relation [2].

*2. Remaining part: $\mathcal{F}_3^\alpha$, $\mathcal{F}_4^\alpha$ and $\mathcal{F}_5^\alpha$*

Now, let us consider the remaining part ${}^1\widehat{T}_{\{\mu\nu\}}(q)$ of the symmetric Compton operator which will vanish if forward matrix elements are taken. However, since the combinations are given in terms of $\mathcal{F}_1^\alpha$, $\mathcal{F}_2^\alpha$ and the combinations $\mathcal{V}_{0\alpha}$ and $\mathcal{V}_{1\alpha}$, modulo a term which vanishes according to the relations (28), we have not to introduce new structure functions. For that part of the Compton operator, taking into account the relation $[1+x^2\hat{p}^2/Q^2]/(x^2/Q^2)\equiv(\hat{p}^{\rm T})^2$, we obtain:

$$
\begin{aligned}
\mathrm{Im}\,{}^1\widehat{T}^{\rm tw2}_{\{\mu\nu\}}(q) &= \mathrm{Im}\,{}^1\widehat{T}^{+}_{\{\mu\nu\}}(q) + \mathrm{Im}\,{}^1\widehat{T}^{-}_{\{\mu\nu\}}(q)\;, \\
\mathrm{Im}\,{}^1\widehat{T}^{\pm}_{\{\mu\nu\}}(q) &= \pi\int_{\mathcal{H}^3}\frac{\mathbf{d}^4\hat{p}}{\kappa^4}\,\delta(\hat{p}^2-1)\Big\{2\Big(\frac{q_\alpha}{q\hat{p}}-\frac{\hat{p}_\alpha}{\hat{p}^2}\Big)\frac{\hat{p}^{\top}_\mu\hat{p}^{\top}_\nu}{(\hat{p}^{\rm T})^2}\,x\,\frac{\hat{p}^2}{Q^2}\,\mathcal{V}_1^\alpha\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big) \\
&\quad + \frac{1}{\hat{p}^2}\Big(\hat{p}^{\top}_\mu g^{\top}_{\nu\alpha} + \hat{p}^{\top}_\nu g^{\top}_{\mu\alpha} - 2\,\frac{\hat{p}^{\top}_\mu\hat{p}^{\top}_\nu}{(\hat{p}^{\top}_\mu)^2}\hat{p}^{\top}_\alpha\Big)\int_0^1\frac{d\tau}{\tau^2}\,\mathcal{W}_2^\alpha\Big(\frac{\xi_\pm}{\tau},x;\frac{\hat{p}}{\kappa}\Big) \\
&\quad + \Big(\frac{q_\alpha}{q\hat{p}}-\frac{\hat{p}_\alpha}{\hat{p}^2}\Big)\Big(g^{\top}_{\mu\nu} - 3\frac{\hat{p}^{\top}_\mu\hat{p}^{\top}_\nu}{(\hat{p}^{\rm T})^2}\Big)\Big[\int_0^1\frac{d\tau}{\tau^2}\Big(\mathcal{W}_1^\alpha\Big(\frac{\xi_\pm}{\tau},x;\frac{\hat{p}}{\kappa}\Big) + \frac{1}{x}\frac{q\hat{p}}{\hat{p}^2}\,\mathcal{W}_2^\alpha\Big(\frac{\xi_\pm}{\tau},x;\frac{\hat{p}}{\kappa}\Big)\Big) \\
&\quad + 2\int_0^1\frac{d\tau}{\tau^2}\Big(\mathcal{V}_0^\alpha\Big(\frac{\xi_\pm}{\tau},x;\frac{\hat{p}}{\kappa}\Big) + x\,\frac{\hat{p}^2}{Q^2}\,\mathcal{V}_1^\alpha\Big(\frac{\xi_\pm}{\tau},x;\frac{\hat{p}}{\kappa}\Big)\Big) + x\,\frac{\hat{p}^2}{Q^2}\,\mathcal{V}_1^\alpha\Big(\xi_\pm,x;\frac{\hat{p}}{\kappa}\Big)\Big]\Big\}\,.
\end{aligned}
\tag{68}
$$

Despite being quite complicated, this piece of the imaginary part of the symmetric Compton operator – which does not contribute if forward matrix elements are taken – is entirely given by $\mathcal{W}_i^\alpha(\xi_\pm, x; \hat{p}/\kappa)$ and $\mathcal{V}_i^\alpha(\xi_\pm, x; \hat{p}/\kappa)$.

C) Operator relations between $\mathcal{V}_i^\alpha$ and $\mathcal{W}_i^\alpha$

Up to now we found the operator analogue (65) of the Callan-Gross relation. Now let us show that $\mathcal{V}_0^\alpha$ and $\mathcal{V}_1^\alpha$ can be expressed by two of the structure functions $\mathcal{W}_i^\alpha$. Namely, taking the trace of Eq. (64) and comparing it with the expression (37) one finds

$$2\mathcal{V}_0^\alpha(\xi_\pm, x; \hat{p}) = \mathcal{W}_L^\alpha(\xi_\pm, x; \hat{p}) - 2\mathcal{W}_1^\alpha(\xi_\pm, x; \hat{p})\,. \tag{69}$$

Furthermore, taking the trace of expression (68) one finds $2\pi\, x\, (\hat{p}^2/Q^2)\, \mathcal{V}_1^\alpha(\xi_\pm, x; \hat{p}/\kappa)$, i.e., the remaining part of (37), as it should be. Now, according to the definition (36), making use of Eqs. (31) – (34) and of expression (67), we get

$$\begin{aligned}
\mathcal{V}_{1\alpha}(\xi_\pm, x; \hat{p}) \;&=\; x\frac{\partial}{\partial x}\frac{x\Omega_\alpha^{(1)}(\xi_\pm, \hat{p})}{\sqrt{1 + x^2\hat{p}^2/Q^2}} = \mp\, x\frac{\partial}{\partial x}\left[\frac{x}{\xi_\pm}x\frac{\partial}{\partial x}\Omega_\alpha^{(2)}(\xi_\pm, \hat{p})\right] \\
&=\; \pm\, x\frac{\partial}{\partial x}\frac{Q^2}{\hat{p}^2}\left[\frac{\sqrt{1 + x^2\hat{p}^2/Q^2}}{x}\mathcal{W}_{L\alpha}(\xi_\pm, x; \hat{p}) - \frac{x\xi_\pm\, \hat{p}^2/Q^2}{\sqrt{1 + x^2\hat{p}^2/Q^2}}\int_x \frac{\mathrm{d}y}{y^2}\, \mathcal{W}_{L\alpha}(\xi_\pm(y), y; \hat{p})\right]\,,
\end{aligned} \tag{70}$$

so that, observing finally the equality $\pm\sqrt{1 + x^2\hat{p}^2/Q^2} = 1 + x\xi_\pm\hat{p}^2/Q^2$ again, we obtain

$$\begin{aligned}
x\frac{\hat{p}^2}{Q^2}\mathcal{V}_{1\alpha}(\xi_\pm, x; \hat{p}) \;&=\; [1 + x\xi_\pm\hat{p}^2/Q^2]\,\Bigg[\, x\frac{\partial}{\partial x}\,\mathcal{W}_{L\alpha}(\xi_\pm, x; \hat{p}) - \frac{1 - x\xi_\pm\hat{p}^2/Q^2}{1 + x^2\hat{p}^2/Q^2}\mathcal{W}_{L\alpha}(\xi_\pm, x; \hat{p}) \\
&\qquad - \frac{x^3\hat{p}^2/Q^2}{[1 + x^2\hat{p}^2/Q^2]^2}\int_x \frac{\mathrm{d}y}{y^2}\,\mathcal{W}_{L\alpha}(\xi_\pm(y), y; \hat{p})\Bigg]\,.
\end{aligned} \tag{71}$$

With these results we find that the structure of the imaginary part of the symmetric twist-2 Compton operator is determined completely by only two operator valued structure functions, namely $\mathcal{W}_{1\alpha}(\xi_\pm, x; \hat{p})$ and $\mathcal{W}_{L\alpha}(\xi_\pm, x; \hat{p})$.

## VI. CONCLUSIONS

The investigation of quantities of definite geometric twist is continued by consideration of the 'complete' twist-2 Compton operator, i.e. given off-cone and taking into account subtraction of all trace terms. We have shown that it has a hidden structure which allows to derive *without further approximations* and *without relation to higher (geometric) twist* Compton operators an operator analog of the Wandzura-Wilczek relation (50) and of the (target mass corrected) Callan-Gross relation (65); additional structural relations (51), (69) and (71), up to now unobserved, occur as well. According to their derivation one is led to conclude that our version of the Wandzura-Wilczek and Callan-Gross relations – also in the case non-forward scattering – are determined completely geometrically, i.e., solely by the twist structure of the operator behind the scattering amplitudes.

All these relations – together with the operator expressions (16), (41) and (55) of the trace, antisymmetric and symmetric part of the entire twist-2 Compton operator – contain the *complete power corrections* resp. *target mass corrections* of the twist-2 Compton operator. The proof of all these relations goes, with more or less obvious changes, along the same lines as has been done for the non-forward virtual Compton amplitude in Ref. [1]. Taking matrix elements of the Compton operator these structural relations remain intact and lead to the corresponding relations for the generalized structure functions and distribution amplitudes. Obviously, they hold equally well for the forward [11] and non-forward scattering amplitudes [10], but also more complicated matrix elements, e.g., when a meson occurs in the final state [12] or when diffractive scattering [13] is considered.

A remarkable result we found was that only the *three independent operator valued structure functions*, namely $\mathcal{G}_{1\alpha}(\xi_\pm, x; \hat{p})$, $\mathcal{W}_{1\alpha}(\xi_\pm, x; \hat{p})$ and $\mathcal{W}_{L\alpha}(\xi_\pm, x; \hat{p})$, completely determine the structure of the imaginary part of the twist-2 Compton operator. In the course of that calculations it was essential to introduce analogs of Bjorken and Nachtmann variables $x$ and $\xi$, respectively, together with a scaling variable $t$. This latter variable measures, when matrix elements are taken, the collinear momentum of the corresponding scattering process, i.e., that part of the momentum pointing in the direction $\mathcal{P} \equiv \sum_i P_i$. Furthermore, let us mention, that $x \to Q^2/q\mathcal{P}$ and that the essential $Q^2-$dependence appears only through $\hat{p}^2/Q^2 \to \mathcal{P}^2/Q^2$, where the arrow shows the corresponding dependence of the matrix elements.

Concerning phenomenological applications our expressions for the entire Compton operator contain mass corrections for the standard deep inelastic scattering and reproduce the existing results. They are also applicable to many different exclusive processes like DVCS and, more general, $e^- + N = e^- + N +$ particle 1 + particle 2 + ... + particle $n$ as well

as $e^+ + e^-$ = particle 1 + particle 2 + ... + particle $n$, thereby using the kinematics of generalized Bjorken region and forming corresponding matrix elements of the Compton operator. Of course at present current conservation remains an open problem which restricts that applicability. It seems to be likely that this can be resolved, at least approximate, by including a center coordinate into the definition of the Compton operator and, as a consequence, adding kinematical twist-3 contributions analogous to the procedures in Refs. [21, 26, 27, 28] or by performing a 'quark spin rotation' as proposed in [24] or using 'rotational invariance' as proposed in [23]. However, this has not been considered here. Especially, because of the complexity of the (geometric) twist-3 Compton operator the study of its structural relations has to be postponed.

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